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Feasibility Study of Reverse Trilateration Strategy with a single Tx for VLP

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ABSTRACT

This paper describes a visible light position (VLP) system relying on a single transmitter to infer position information. The system adopts a reverse trilateration scheme, where a set of three photodiodes is employed to estimate the position. This system is suitable for low power and low complexity systems. The position information is inferred through Receiver signal strength.

SYSTEM DESIGN AND MODELING

The schematic of the proposed reversed trilateration strategy is illustrated in Fig. 1. The VLP sensor comprises 3 PDs positioned on a circle of radius R_s. Let the positions of k^{th} PD be $\boldsymbol{X}^o_k = (x^o_k, y^o_k)^T$. The Tx projection on the x-y plane is $\boldsymbol{X_T} = (x_T, y_T)^T$. The position of the VLP sensor in the x-y plane as a function of an arbitrary translation $\boldsymbol{X} = (x, y)^T$ and an arbitrary rotation $\boldsymbol{R}(\theta)$. The estimated position is directly given by:

$$\boldsymbol{X} = \boldsymbol{X_T} + \frac{1}{2R_s}\boldsymbol{R}(\theta)\boldsymbol{A_s}^{-1}\Delta\boldsymbol{r}, \boldsymbol{A_s} = \frac{1}{R_s}\begin{bmatrix}(\boldsymbol{X}_2^o)^T - (\boldsymbol{X}_1^o)^T \\ (\boldsymbol{X}_3^o)^T - (\boldsymbol{X}_2^o)^T\end{bmatrix}, \Delta\boldsymbol{r} = \begin{bmatrix} r_2^2 - r_1^2 \\ r_3^2 - r_2^2 \end{bmatrix}. \quad (1)$$

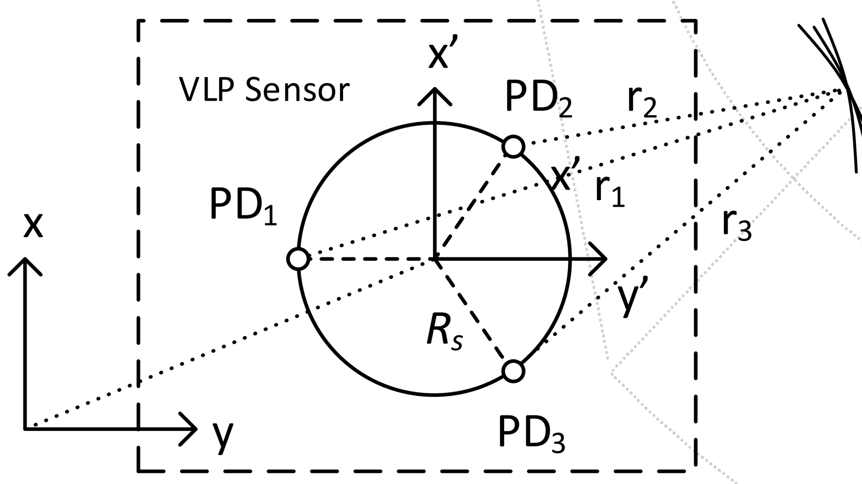

Fig.1. Reverse trilateration concept.

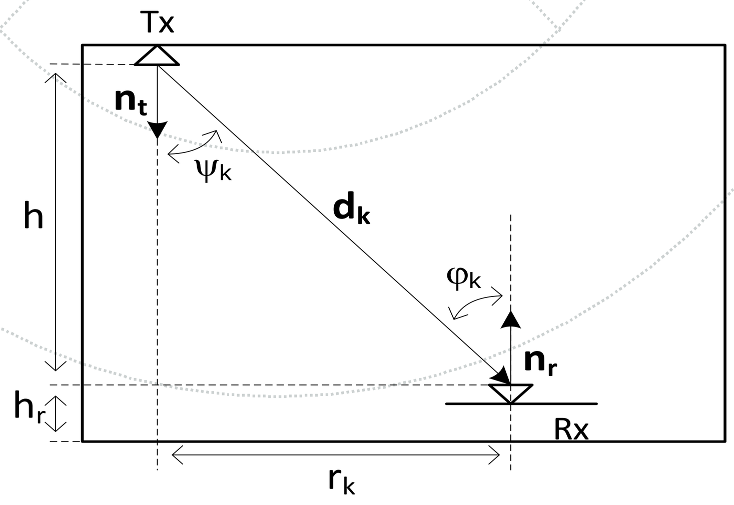

Fig.2. System model of a single LED-based Tx and the Rx sensor with three PDs.

Fig. 2 depicts the set up conditions for signal transmission, between the Tx and the k^{th} Rx. The received signal for Lambertian emitter for a line of sight (LoS) path is given by

$$S_k = H_o T_s(\varphi_k) g(\varphi_k)\frac{cos^m(\psi_k)cos(\varphi_k)}{\|\boldsymbol{d_k}\|^2} \quad (2)$$

where

$$H_o = \frac{m+1}{2\pi}G_o\mathcal{R}A_r P_t, \qquad m = -\frac{\ln(2)}{ln(cos(HPA))} \quad (3)$$

where ψ_k is the angle between $\boldsymbol{d_k}$ and the LED normal, φ_k is the angle between $\boldsymbol{d_k}$ and the PD normal. $T_s(\varphi_k)$ is the Rx filter and $g(\varphi_k)$ is the optical concentrator. For the forgoing analysis, we will assume that, $T_s(\varphi_k)$ and $g(\varphi_k)$ are both unity. Knowing that $||\boldsymbol{d_k}^2|| = r_k^2 + h^2$ we can solve (2) to find r_k^2 as given by:

$$r_k^2 = \left(\frac{H_o h^{m+1}}{S_k}\right)^{\frac{2}{m+3}} - h^2 \quad (4)$$

where h is the vertical distance between the Tx and the Rx.

ERROR PERFORMANCE ANALYSIS

The error performance analysis can be assessed using (1). If the detected signal is affected by noise, the measured signal power will contain errors, which are defined by small changes in $\boldsymbol{\Delta r}$, i.e., $\boldsymbol{\Delta r} \rightarrow \boldsymbol{\Delta r} + \delta\boldsymbol{r}$. Using (1) we have $\boldsymbol{\Delta X} \rightarrow \boldsymbol{\Delta X} + \delta\boldsymbol{X}$, where $\delta\boldsymbol{X}$ is the coordinate displacement due to $\delta\boldsymbol{r}$, which is given by:

$$\boldsymbol{\delta X} = \frac{1}{2R_s}\boldsymbol{R}(\theta)\boldsymbol{A_s}^{-1}\boldsymbol{\delta r}. \quad (5)$$

We may transform this into a distance error, as given by:

$$\boldsymbol{\delta X} = \sqrt{\boldsymbol{\delta X^T \delta X}} = \frac{1}{2R_s}\sqrt{\boldsymbol{\delta r}^T\boldsymbol{\Lambda_s \delta r}}. \quad (6)$$

where $\boldsymbol{\Lambda_s} = (\boldsymbol{A_s}^{-1})^T\boldsymbol{A_s}^{-1}$. The effect of noise on the detected signal, by the k^{th} PD is expressed by $S_k \rightarrow S_k + n_k$, where n_k is the additive white Gaussian noise. Using the signal to noise ratio (SNR), γ, the coordinate displacement including noise can be expressed as

$$\boldsymbol{\delta X} = G_S\frac{(H_o h^{m+1})^{\frac{2}{m+3}}}{2R_s}\left(\left(1+\frac{1}{\sqrt{\gamma}}\right)^{-\frac{2}{m+3}} - 1\right) \quad (7)$$

where $G_s = (\Delta_s^T\Lambda_s\Delta_s)^{1/2}$. Fig. 3 depicts G_s for two different values of HPA (i.e., 60° and 30°), in a room of size 10x10x2.4 m^3, with one Tx positioned in the center.

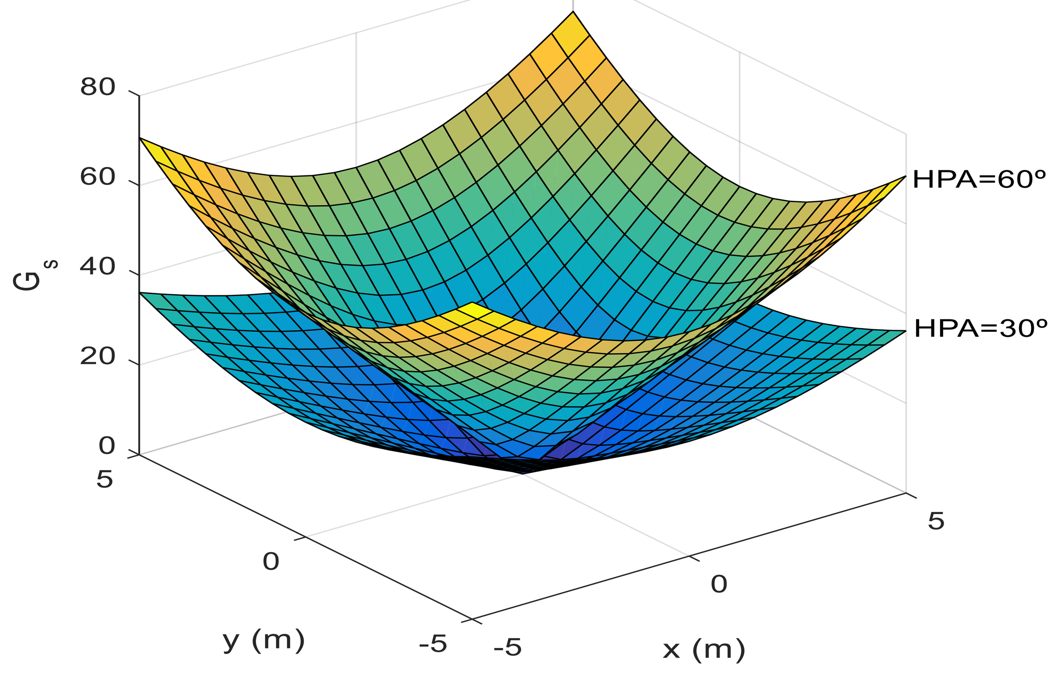

Fig. 3. The distance error geometrical factor G_s for two different HPA values.

SIMULATION RESULTS

Table 1 presents the system parameters. Figs. 4 and 5 depict the PE spatial distributions in a room of 10×10×2.4 m3, for SNRs of 50 and 60 dB, respectively. Figs. 6 and 7 depict the PE dependency on SNR and Rs. Figs. 8 and 9 depicts the PE dependency on both SNR and HPA. Fig. 10 depict the averaging effect on the PE. It can be seen from the Fig 10 that, the error performance increases with the number of averaging steps and SNR. Therefore, it is possible to use averaging to improve PE. In a total of N estimates, let #(err < ε) represent the number of estimates with the positioning error below ε, the PE is

Table 1. Default value of the system parameters.

Tx-Rx parameter		
A_r	100 mm²	Area of the PD
$\mathcal{R}$	1 A/W	Responsivity
P_t	1 W	Transmit power
G_o	1 MΩ	Rx's gain
HPA	60º	Half power angle
dx	0.5 m	x grid resolution
dy	0.5 m	y grid resolution
n_t	$(0, 0, -1)^T$	TX heading vector
n_r	$(0, 0, 1)^T$	RX heading vector
X_T	$(0, 0, h + h_r)^T$	TX position
R_s	0.1 m	Sensor radius
h	2.2 m	Ceiling height
h_r	0.2 m	Sensor height
θ	0º	Sensor rotation angle

$$e_{rr}(\varepsilon) = \frac{(e_{rr} < \varepsilon)}{N} \quad (8)$$

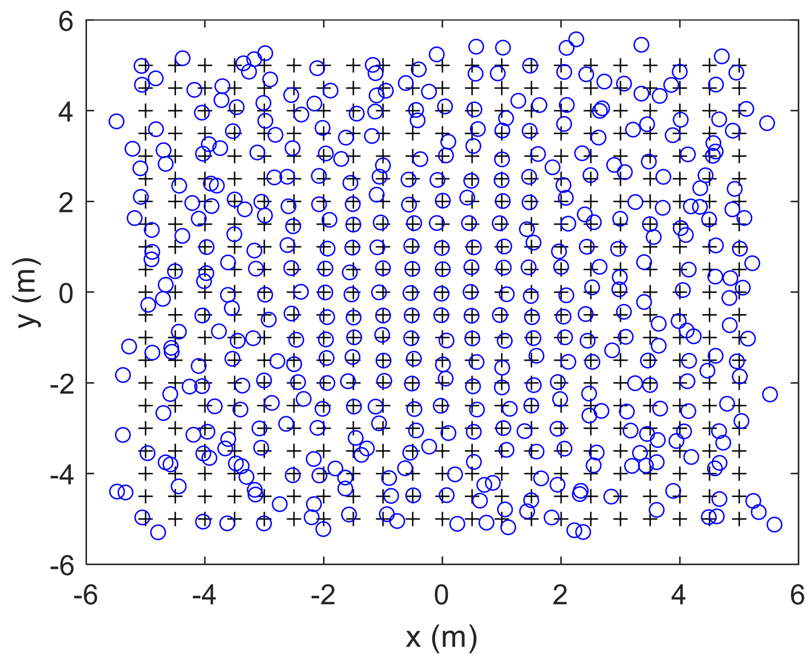

Fig.4. The positioning error spatial distribution with SNR=50 dB.

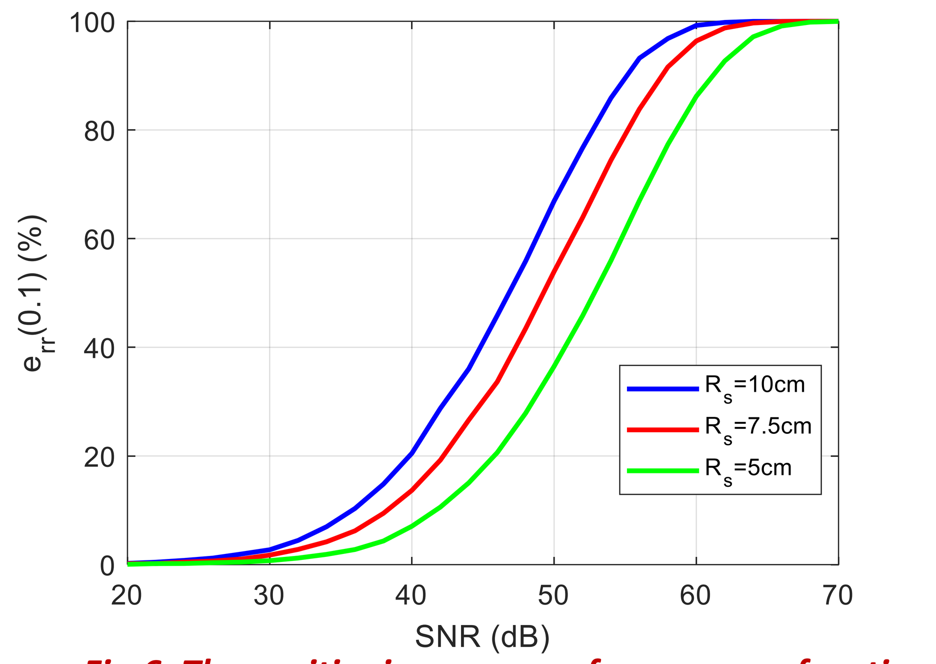

Fig.6. The positioning error performance as function of SNR, for fixed values of R_s.

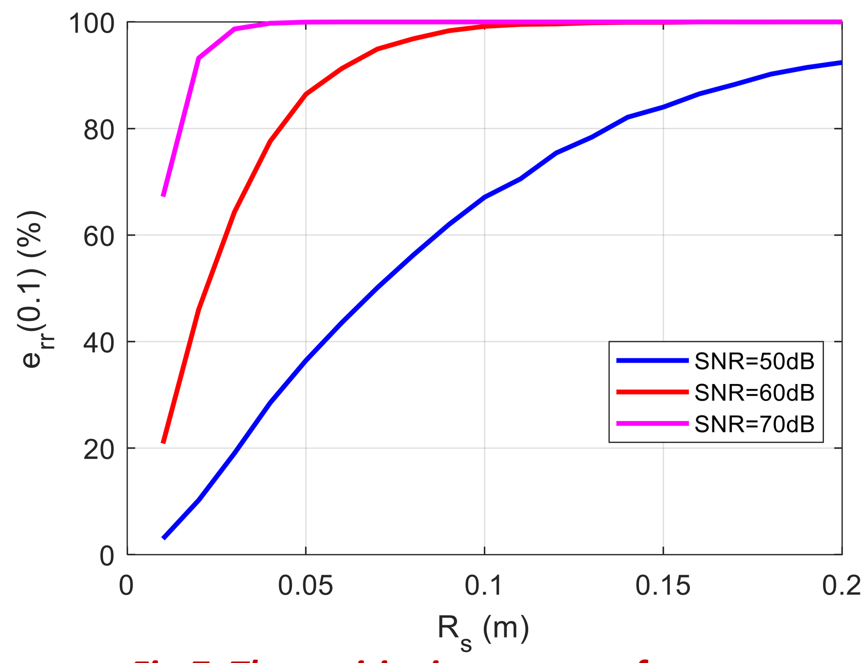

Fig.7. The positioning error performance as function of R_s, for fixed values of SNR.

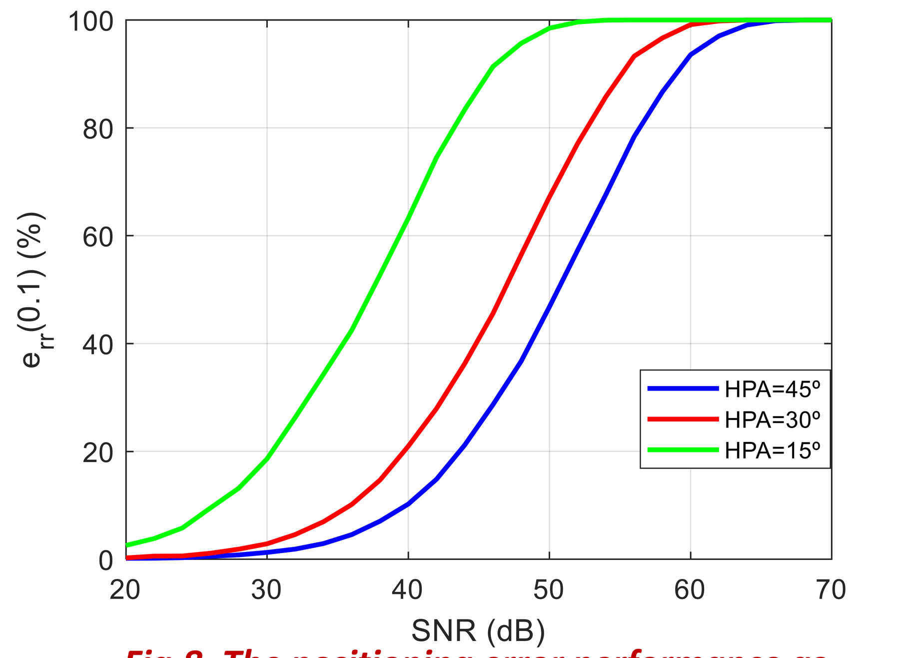

Fig.8. The positioning error performance as function of SNR, for fixed values of HPA.

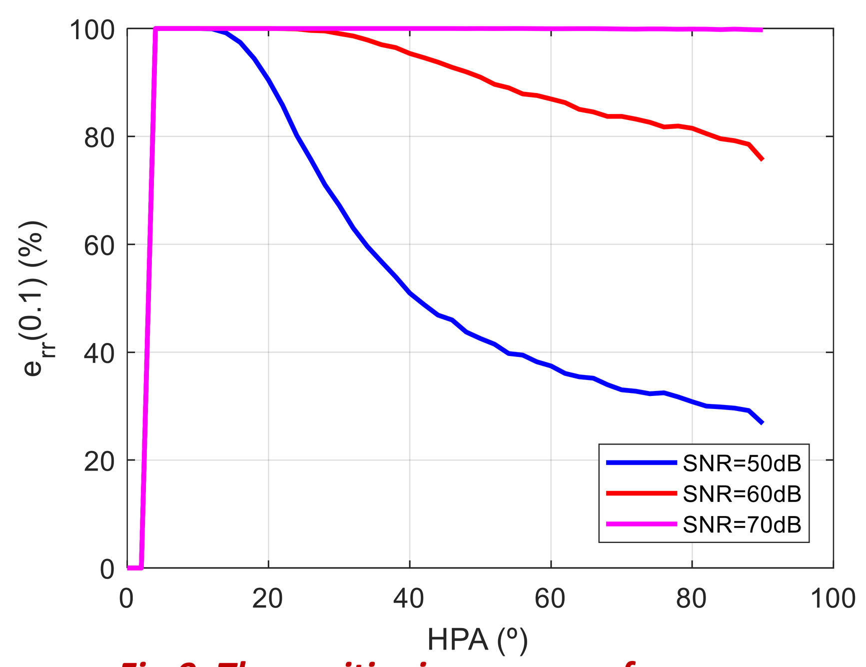

Fig.9. The positioning error performance as function of HPA, for fixed values of SNR.

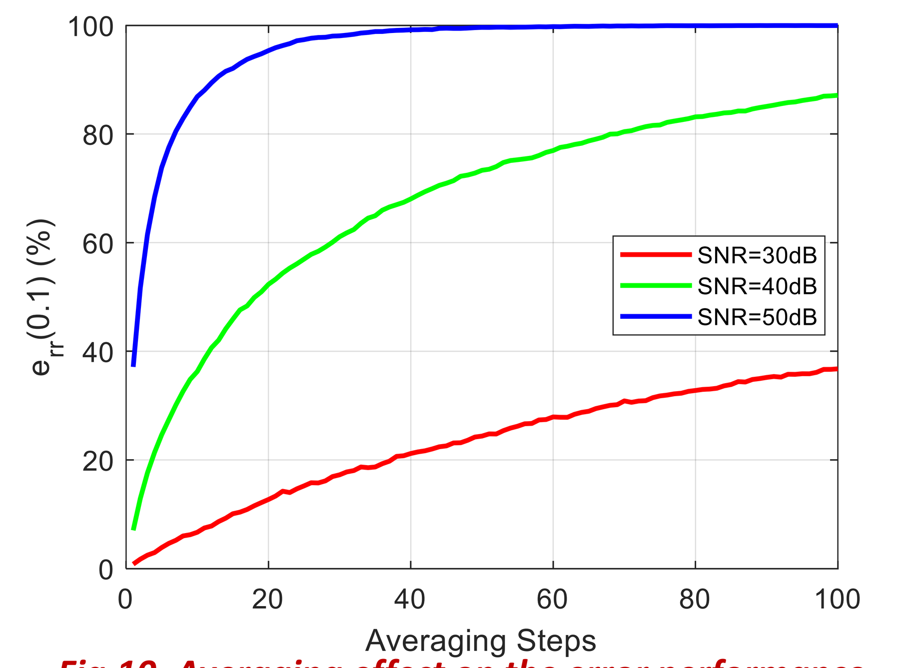

Fig.10. Averaging effect on the error performance for a range of SNRs.

CONCLUSION

This paper presented a feasibility study on the usage of reverse trilateration for position estimation, based on RSS. Simulation results disclosed that, the system is very susceptible to the noise, demanding very high SNR in order to achieve low positioning error performance. Considering that RSS can be supported with averaging techniques able to improve SNR, the proposed system can represent a possible choice for low power and low complexity VLP positioning sensors.

ACKNOWLEDGEMENT

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